Einstein coefficients

Summary of equations from video on Einstein coefficients.

We assume a two-level system with states 1 and 2. We denote N_1 (N_2), $E_1(E_2)$ and $g_1(g_2)$ the number of molecules occupying state 1 (2), the energy of state 1 (2) and the degeneracy of state 1 (2), respectively.

Density of light energy

$$\bar{W}(\omega) = W_T(\omega) + W_E(\omega), \tag{1}$$

kinetic equations

$$-\frac{d}{dt}N_1 = \frac{d}{dt}N_2 = -N_2A_{21} + N_1\bar{B}_{12} - N_2\bar{B}_{21}. \tag{2}$$

Coefficient of absorption

$$\bar{B}_{12} = B_{12}\bar{W}(\omega), \tag{3}$$

coefficient of stimulated emission

$$\bar{B}_{21} = B_{21}\bar{W}(\omega), \tag{4}$$

coefficient of spontanious emission is A_{21}.

In equilibrium without external sources

$$-N_2A_{21} + (N_1B_{12} - N_2B_{21})\,W_T(\omega) = 0. \tag{5}$$

We get

$$W_T(\omega) = \frac{A_{21}}{\left(\frac{N_1}{N_2}\right)B_{12} - B_{21}}. \tag{6}$$

Detailed balance requires

$$\frac{N_1}{N_2} = \left(\frac{g_1}{g_2}\right)e^{\frac{\hbar\omega}{k_BT}}. \tag{7}$$

Planck's law reads

$$W_T(\omega) = \frac{\hbar\omega^3}{\pi^2c^3}\frac{1}{e^{\frac{\hbar\omega}{k_BT}} - 1} \tag{8}$$

Final equations

$$\left(\frac{g_1}{g_2}\right)B_{12} = B_{21}, \tag{9}$$

$$\frac{A_{21}}{B_{21}} = \frac{\hbar\omega^3}{\pi^2c^3}. \tag{10}$$