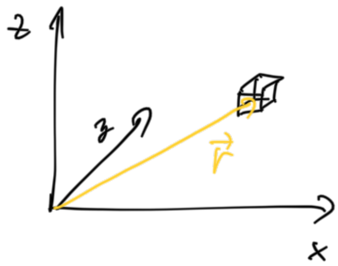
Popis kvantových systémů v prostoru



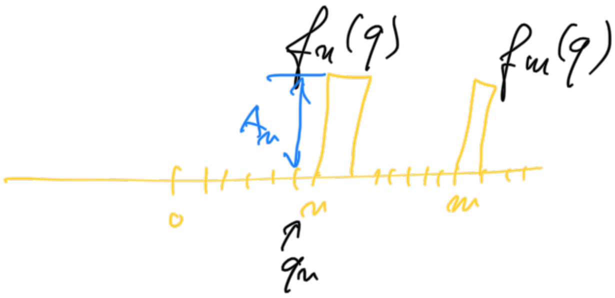
$$\hat{\vec r}\,|\vec r\rangle = \vec r\,|\vec r\rangle$$

Zase do 1D

$$q\,,\ |q\rangle\,,\ \hat q$$

Chceme vymyslet reprezentaci, pomocí které bychom v prostoru popsali $|q_n\rangle$ (n-té místo na ose q) tak, aby

$$\langle q_n|q_m\rangle = \delta_{nm}$$



skalární součin

$$\int_{-\infty}^{\infty} dq\, f_n(q)\, f_m(q) = \delta_{nm}\int_{q_n}^{q_n+\Delta q} dq\, A_n^2 = \delta_{nm} A_n^2 \Delta q$$

$$\Rightarrow A_n^2 \Delta q = 1$$

$$A_n = \frac{1}{\sqrt{\Delta q}}$$

$$f_n(q) = \frac{1}{\sqrt{\Delta q}}\,\Theta(q-q_n)\,\Theta(-(q-(q_n+\Delta q)))$$

q_n

$$\langle q_n|q_m\rangle = \int_{-\infty}^{\infty} dq\, f_n^*(q)\, f_m(q) = \delta_{nm}$$

$\Delta q \to 0$

$$f_n^{(\Delta q)}(q') \to f_{q_n}^{(\Delta q)}(q') \xrightarrow{\Delta q \to 0} f_q(q') = \delta(q'-q)$$

Operátor souřadnice

$$\hat q = \int dq\, q\, |q\rangle\langle q|$$

$$|q\rangle \longrightarrow f_q(q') = \delta(q'-q)$$

$$\langle \bar q|q\rangle = \int_{-\infty}^{\infty} dq'\, \delta(q'-\bar q)\,\delta(q'-q) = \delta(\bar q - q)$$

$$\boxed{\hat q\,|q'\rangle} = \int dq\, q\, |q\rangle \underbrace{\langle q|q'\rangle}_{\delta(q-q')} = \boxed{q'\,|q'\rangle}$$