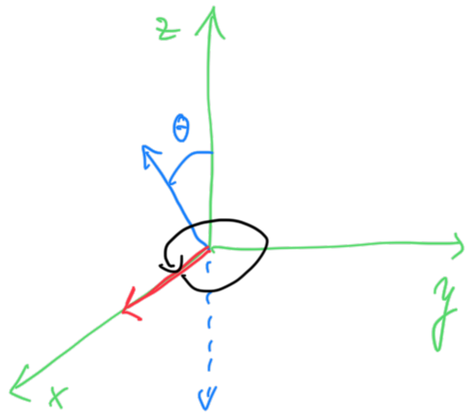
Dvouhladinový systém

1) Spin
2) atomy a molekuly
3) Qbit

$$|\psi\rangle = a|0\rangle + b|1\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$\theta, \varphi$$

Blochova sféra



$\theta = 0 \rightarrow |0\rangle$ – osa R +

$\theta = \pi \rightarrow |1\rangle$

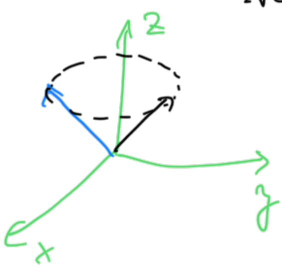
— osa R —

$\left.\begin{array}{l}\varphi \ldots \text{od osy } x \;;\; \varphi = 0 \\ \theta = \frac{\pi}{2}\end{array}\right\} |\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

$$\varepsilon_2 - \varepsilon_1 = \hbar\omega_{21}$$

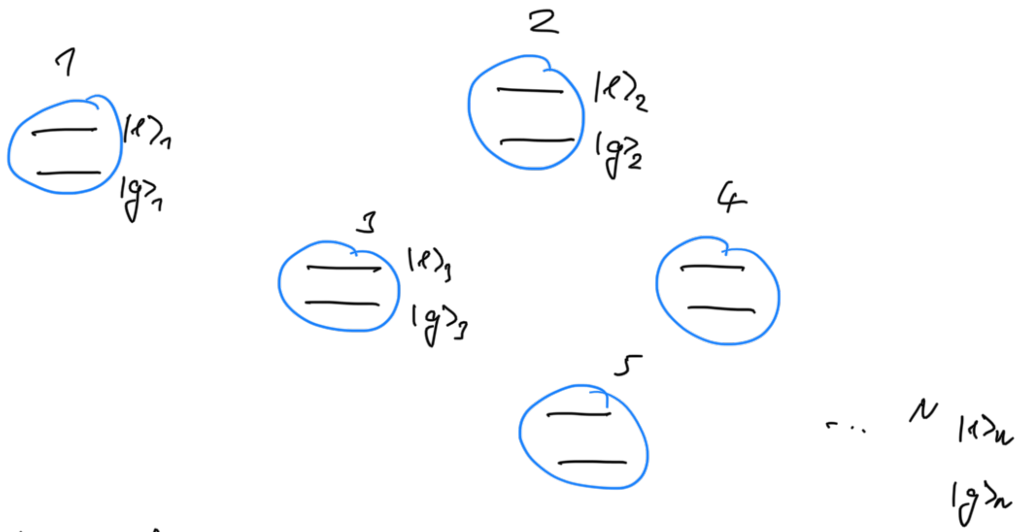
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{-i\omega_{21}t}|1\rangle\right)$$

$|\psi(0)\rangle = a|0\rangle + b|1\rangle \longrightarrow \left.\begin{array}{l}\cos\frac{\theta}{2} = a \\ \sin\frac{\theta}{2} = b\end{array}\right\} \Rightarrow \tan\frac{\theta}{2} = \frac{b}{a}$



$$|\psi(t)\rangle = a|0\rangle + b\,e^{-i\omega_{21}t}|1\rangle$$

Systém mnoha dvouhladinových systémů



Hamiltonián

$$H = \sum_n H_n \qquad\qquad H_n = \varepsilon_g^{(n)} |g\rangle_n \langle g|_n + \varepsilon_e^{(n)} |e\rangle_n \langle e|_n$$

$$= H_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_3 \otimes \cdots \otimes \mathbb{1}_n + \mathbb{1}_1 \otimes H_2 \otimes \mathbb{1}_3 \cdots \otimes \mathbb{1}_N + \dots$$

Báze stavů např:

$$|g\rangle_1 |e\rangle_2 |g\rangle_3 \dots |e\rangle_N \qquad g \Rightarrow 0$$
$$e \Rightarrow 1$$

$$= |0\rangle_1 |1\rangle_2 |0\rangle_3 \dots |1\rangle_N$$

$$\equiv |010 \dots 1\rangle$$

Př:

$$\langle 01100 | H | 11100 \rangle = 0$$

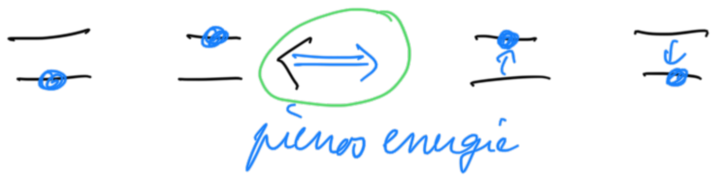
Kolik má systém N dvouhladinových systémů stavů?

$$\Rightarrow n_{tot} = 2^N$$

počet stavů roste exponenciálně

Párová interakce

stav $|0\rangle_1|1\rangle_2 \longrightarrow |1\rangle_1|0\rangle_2$



$$H = \begin{array}{c} \\ |0\rangle|0\rangle \\ |1\rangle|0\rangle \\ |0\rangle|1\rangle \\ |1\rangle|1\rangle \end{array} \begin{array}{c} \begin{array}{cccc} |0\rangle|0\rangle & |1\rangle|0\rangle & |0\rangle|1\rangle & |1\rangle|1\rangle \end{array} \\ \begin{pmatrix} \varepsilon_g^{(1)}+\varepsilon_g^{(2)} & 0 & 0 & 0 \\ 0 & \varepsilon_e^{(1)}+\varepsilon_g^{(2)} & J & 0 \\ 0 & J & \varepsilon_g^{(1)}+\varepsilon_e^{(2)} & 0 \\ 0 & 0 & 0 & \varepsilon_e^{(1)}+\varepsilon_e^{(2)} \end{pmatrix} \end{array}$$

0 1 2

Prostor stavů se rozpadá na „páry" s fixním počtem excitací

- neexcitovaný stav
- 1-excitované stavy
- 2- —"—

⋮

- N-excitované stavy

V 1-excitovaném páru (bloku, manifoldu) roste počet stavů pouze lineárně.

$\boxed{\text{Qbit}} \rightarrow 2^N$ stavů

Časový vývoj systému se dvěma stavy

$|1\rangle\ ,\ \varepsilon_1$

Interakční energie J

$|2\rangle\ ,\ \varepsilon_2$

$$\hat{H} = \varepsilon_1 |1\rangle\langle 1| + \varepsilon_2 |2\rangle\langle 2| + J|2\rangle\langle 1| + J|1\rangle\langle 2|$$

$$E_1 = \langle 1|H|1\rangle = \varepsilon_1$$

$$H = \begin{pmatrix} \varepsilon_1 - \lambda & J \\ J & \varepsilon_2 - \lambda \end{pmatrix} \Rightarrow$$

Schrödingerova rovnice

$$\boxed{\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{i}{\hbar} H |\psi(t)\rangle}$$

$$\varepsilon_1 \neq \varepsilon_2$$

Diagonalizace hamiltoniánu

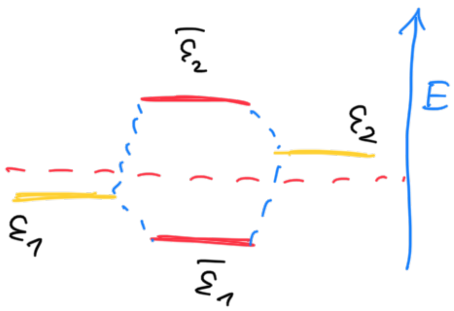
$$\det(H - \lambda \mathbb{1}) = 0$$

$$(\varepsilon_1 - \lambda)(\varepsilon_2 - \lambda) - J^2 = 0$$

$$\lambda^2 - (\varepsilon_1 + \varepsilon_2)\lambda + \varepsilon_1\varepsilon_2 - J^2 = 0$$

$$\lambda_{1,2} = \frac{\varepsilon_1 + \varepsilon_2 \pm \sqrt{(\varepsilon_1 + \varepsilon_2)^2 - 4(\varepsilon_1\varepsilon_2 - J^2)}}{2}$$

$$\bar{\varepsilon}_{1,2} = \frac{\varepsilon_1 + \varepsilon_2}{2} \pm \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4J^2}$$



Vlastní vektory – normované a ortogonální

$$|\xi\rangle = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \qquad \cos^2\phi + \sin^2\phi = 1$$

$$\begin{pmatrix} \varepsilon_1 & J \\ J & \varepsilon_2 \end{pmatrix} \begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix} = \bar{\varepsilon}_1 \begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix}$$

$$\boxed{\varepsilon_1 \cos\phi - J \sin\phi = \bar{\varepsilon}_1 \cos\phi}$$

$$(\varepsilon_1 - \bar{\varepsilon}_1)\cos\phi - J\sin\phi = 0$$

$J > 0$
$\varepsilon_1 > \bar{\varepsilon}_1$
$\phi > 0$ a spíše malý

$$\boxed{J\cos\phi - \varepsilon_2 \sin\phi = -\bar{\varepsilon}_1 \sin\phi}$$

$$\varepsilon_1 \cos\phi \sin\phi - \varepsilon_2 \sin\phi\cos\phi + J(\sin^2\phi + \cos^2\phi) = 0$$

$$J(\cos^2\phi - \sin^2\phi) = (\varepsilon_2 - \varepsilon_1)\cos\phi\sin\phi$$

$$\frac{\cos\phi\sin\phi}{\cos^2\phi - \sin^2\phi} = \frac{J}{\varepsilon_2 - \varepsilon_1}$$

$$\left.\begin{aligned} \sin\phi\cos\phi &= \tfrac{1}{2}\sin 2\phi \\ \sin^2\phi - \cos^2\phi &= -\cos 2\phi \end{aligned}\right\} \Rightarrow \quad \frac{1}{2}\tan 2\phi = \frac{J}{\varepsilon_2 - \varepsilon_1}$$

$$\boxed{\phi = \frac{1}{2}\arctan\frac{2J}{\varepsilon_2 - \varepsilon_1}}$$

Mixing angle

$$|\varepsilon_1\rangle = \begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix} \qquad |\varepsilon_2\rangle = \begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix}$$

Evoluční operátor systému se dvěma stavy

$$H = \begin{pmatrix} \varepsilon_1 & J \\ J & \varepsilon_2 \end{pmatrix} \Rightarrow |\xi_1\rangle = \begin{pmatrix} \cos\phi \\ -\sin\phi \end{pmatrix} ; \ |\xi_2\rangle = \begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix}$$

$$\bar{\varepsilon}_1 = \frac{\varepsilon_1 + \varepsilon_2}{2} - \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4J^2}$$

$$H' = S^{-1} H S$$

$$\bar{\varepsilon}_2 = \frac{\varepsilon_1 + \varepsilon_2}{2} + \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 4J^2}$$

$$U(t,t_0) = \exp\left(-\frac{i}{\hbar} H (t - t_0)\right) = S \exp\left(-\frac{i}{\hbar} S^{-1} H S\right) S^{-1}$$

$$\boxed{U(t,t_0)} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} e^{-\frac{i}{\hbar}\bar{\varepsilon}_1 (t-t_0)} & 0 \\ 0 & e^{-\frac{i}{\hbar}\bar{\varepsilon}_2 (t-t_0)} \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} c\phi & s\phi \\ -s\phi & c\phi \end{pmatrix} \begin{pmatrix} e^{-\frac{i}{\hbar}\bar{\varepsilon}_1 (t-t_0)} \cos\phi & ; -e^{-1} \sin\phi \\ e^{-2} \sin\phi & ; e^{-2} \cos\phi \end{pmatrix} =$$

$$= \boxed{\begin{pmatrix} e^{-\frac{i}{\hbar}\bar{\varepsilon}_1 (t-t_0)} \cos^2\phi + e^{-\frac{i}{\hbar}\bar{\varepsilon}_2 (t-t_0)} \sin^2\phi & ; \left(e^{-\frac{i}{\hbar}\bar{\varepsilon}_2 (t-t_0)} - e^{-\frac{i}{\hbar}\bar{\varepsilon}_1 (t-t_0)}\right) s\phi\, c\phi \\ \left(e^{-\frac{i}{\hbar}\bar{\varepsilon}_2 (t-t_0)} - e^{-\frac{i}{\hbar}\bar{\varepsilon}_1 (t-t_0)}\right) \sin\phi \cos\phi & ; e^{-\frac{i}{\hbar}\bar{\varepsilon}_1 (t-t_0)} \sin^2\phi + e^{-\frac{i}{\hbar}\bar{\varepsilon}_2 (t-t_0)} \cos^2\phi \end{pmatrix}}$$

Pravděpodobnost P_1 obsazení stavu $|1\rangle$

$$|\Psi(t_0)\rangle = |\Psi_0\rangle = a_0|1\rangle + b_0|2\rangle = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$

$$a_0 = 1 \quad \Rightarrow$$

$$|\Psi(t)\rangle = U(t,t_0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos^2\phi\, e^{-\frac{i}{\hbar}\bar{\varepsilon}_1(t-t_0)} + \sin^2\phi\, e^{-\frac{i}{\hbar}\bar{\varepsilon}_2(t-t_0)} \\ \sin\phi\cos\phi\left(e^{-\frac{i}{\hbar}\bar{\varepsilon}_2(t-t_0)} - e^{-\frac{i}{\hbar}\bar{\varepsilon}_1(t-t_0)}\right) \end{pmatrix}$$

$$\boxed{P_1(t)} = \langle\Psi(t)|1\rangle\langle 1|\Psi(t)\rangle = |a(t)|^2$$

$$z + z^* = a + \cancel{ib} + a - \cancel{ib} = 2a$$

$$= \left(\cos^2\phi\, e^{\frac{i}{\hbar}\bar{\varepsilon}_1(t-t_0)} + \sin^2\phi\, e^{\frac{i}{\hbar}\bar{\varepsilon}_2(t-t_0)}\right)\left(\text{---} \quad \text{---} \\ \ldots\ldots\right)$$

$$= \cos^4\phi + \sin^4\phi + \sin^2\phi\cos^2\phi\, e^{\frac{i}{\hbar}(\bar{\varepsilon}_2-\bar{\varepsilon}_1)(t-t_0)} + c.c.$$

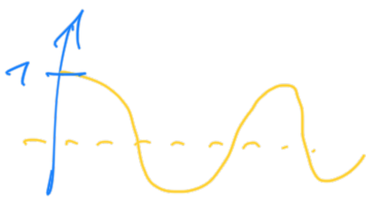
$$= \boxed{\cos^4\phi + \sin^4\phi + 2\sin^2\phi\cos^2\phi\cos(\omega_{21}(t-t_0))}$$

$t = t_0$

$$P_1(t_0) = \cos^4\phi + \sin^4\phi + 2\sin^2\phi\cos^2\phi = (\cos^2\phi + \sin^2\phi)^2 = 1$$

$$P_1\left(t - t_0 = \tfrac{T}{2}\right) = (\cos^2\phi - \sin^2\phi)^2 < 1$$

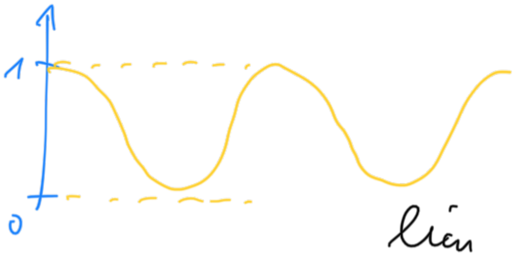
$$\phi = \frac{1}{2}\arctan\left(\frac{2J}{\varepsilon_2 - \varepsilon_1}\right)$$



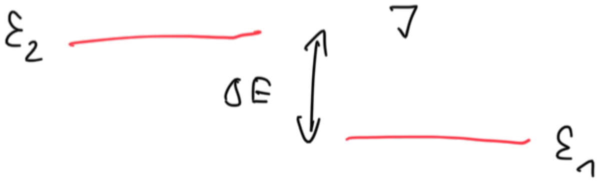
$\sin\phi = \cos\phi$

$\phi = \frac{\pi}{4}$

$\varepsilon_2 = \varepsilon_1$



$\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$



$\Rightarrow J \to \infty$

$\varepsilon_2 \to \varepsilon_1$