Operátor momentu hybnosti

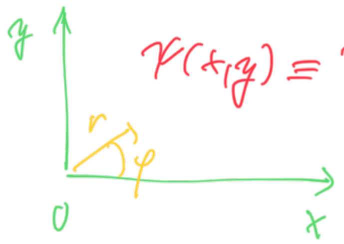
Operátor hybnosti $\hat{p} = -i\frac{\partial}{\partial q}$

$$\Psi(q+a) = e^{a\frac{\partial}{\partial q}}\Psi(q) = e^{+ia\hat{p}}\Psi(q)$$

Co třeba rotace funkce

$x = r\cos\varphi$

$y = r\sin\varphi$



$$\Psi(x,y) \equiv \Psi(r,\varphi) \longrightarrow \Psi(r,\varphi+\varphi_0) = D_{\varphi_0}\Psi(r,\varphi)$$

$$D_{\varphi_0} = e^{\varphi_0\frac{\partial}{\partial\varphi}}$$

$$\frac{\partial}{\partial\varphi}\Psi(x,y) = \lim_{d\varphi\to 0}\frac{\Psi(r\cos(\varphi+d\varphi), r\sin(\varphi+d\varphi)) - \Psi(r\cos\varphi, r\sin\varphi)}{d\varphi}$$

$$= \lim_{d\varphi\to 0}\frac{\Psi(r[\cos\varphi - \sin\varphi\cdot d\varphi], r\sin\varphi + r\cos\varphi\, d\varphi) - \Psi(x,y)}{d\varphi}$$

$$= \lim_{d\varphi\to 0}\frac{\cancel{\Psi(x,y)} + \frac{\partial}{\partial x}\Psi(x, r\sin\varphi\ldots)(\overbrace{-\sin\varphi}^{-y}\, \cancel{d\varphi}) + \frac{\partial\Psi}{\partial y}(x,y)\overbrace{r\cos\varphi}^{x}\,\cancel{d\varphi}\ \cancel{-\Psi}}{\cancel{d\varphi}}$$

$$= \left(\frac{\partial}{\partial x}\Psi(x,y)\right)(-y) + x\frac{\partial}{\partial y}\Psi(x,y) = \left(\vec{x}\times\frac{\partial}{\partial\vec{r}}\right)_z$$

$$\hat{l}_z = \frac{\partial}{\partial\varphi} = x\underbrace{\frac{\partial}{\partial y}}_{i\hat{p}_y} - y\underbrace{\frac{\partial}{\partial x}}_{i\hat{p}_x} \qquad D_{\varphi_0} = e^{\varphi_0\hat{l}_z} = e^{i\varphi_0\hat{L}_z}$$

$$= i(x\hat{p}_y - y\hat{p}_x)$$

$$\boxed{\vec{\hat{L}} = \vec{\hat{r}} \times \vec{\hat{p}}}$$

Moment hybnosti

$$\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$$
$$\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z$$
$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$$

$$\boxed{\hat{L}_x\hat{L}_y - \hat{L}_y\hat{L}_x} = (\hat{y}\hat{p}_z - \hat{z}\hat{p}_y)(\hat{z}\hat{p}_x - \hat{x}\hat{p}_z)$$
$$- (\hat{z}\hat{p}_x - \hat{x}\hat{p}_z)(\hat{y}\hat{p}_z - \hat{z}\hat{p}_y)$$
$$= y p_z z p_x - \cancel{z^2 p_y p_x} - \cancel{y x p_z^2} + z x p_y p_z$$
$$- z p_x y p_z + \cancel{x y p_z^2} + \cancel{z^2 p_x p_y} - x p_z z p_y$$
$$= y p_x (\underbrace{p_z z - z p_z}_{-i\hbar}) + x p_y (\underbrace{z p_z - p_z z}_{+i\hbar})$$
$$= i\hbar(\hat{x}\hat{p}_y - \hat{y}\hat{p}_x) = \boxed{i\hbar \hat{L}_z}$$

$$\boxed{[\hat{L}_l, \hat{L}_m] = i\hbar\, \varepsilon_{lmn} \hat{L}_n}$$

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$ Kvadrát operátoru momentu hybnosti

$$\boxed{[\hat{L}^2, \hat{L}_x]} = \underbrace{[\hat{L}_x^2, \hat{L}_x]}_{=0} + [\hat{L}_y^2, \hat{L}_x] + [\hat{L}_z^2, \hat{L}_x]$$

$$[A^2, B] = A^2B - BA^2$$

$$[L_y, L_x] = -i\hbar L_z$$

$$[L_z, L_x] = +i\hbar L_y$$

$$= L_y [L_y, L_z] + [L_y, L_x] L_y + L_z [L_z, L_x] + [L_z, L_x] L_z$$

$$= -i\hbar \cancel{L_y L_z} - i\hbar \cancel{L_z L_y} + i\hbar \cancel{L_z L_y} + i\hbar \cancel{L_y L_z} = 0$$

$$= \underbrace{AAB - ABA}_{A[A,B]} + \underbrace{ABA - BAA}_{[A,B]A}$$