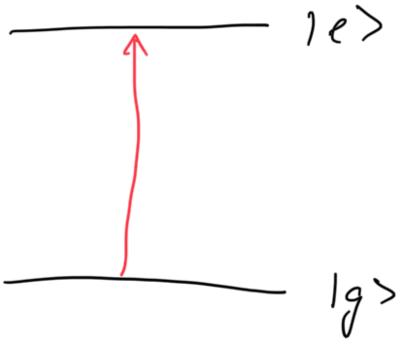
Odezva striktně dvouhladinového systému



$$H_S = \varepsilon_g |g\rangle\langle g| + \varepsilon_e |e\rangle\langle e|$$

$$\omega_{eg} = \frac{\varepsilon_e - \varepsilon_g}{\hbar}$$

$$\underline{\underline{\hat{\mu}}} = d_{eg} |e\rangle\langle g| + d_{ge} |g\rangle\langle e|$$

$$E(t) = E_0 \left(e^{i\omega t} + e^{-i\omega t}\right)$$

Standardní rezonanční podmínka

$$\boxed{\omega \approx \omega_{eg}}$$

Schrödingerova rovnice

$$\frac{\partial}{\partial t} |\psi^{(I)}(t)\rangle = \frac{i}{\hbar} \hat{\mu}^{(I)}(t) |\psi^{(I)}(t)\rangle E(t)$$

$$\alpha(\omega) \approx \frac{\left\langle \frac{\partial}{\partial t} P_e(t) \right\rangle_T}{E_0^2} \leftarrow$$ absorpční spektrum na frekvenci $\underline{\omega}$

$$\frac{\partial}{\partial t} P_e(t) = \frac{\partial}{\partial t} \left(\langle \psi^{(I)}(t) | \underline{e\rangle\langle e} | \psi^{(I)}(t) \rangle \right) =$$

$$= 2\,\mathrm{Re}\left(\frac{\partial}{\partial t}\langle\Psi^{(I)}(t)|e\rangle\right)\langle e|\Psi^{(I)}(t)\rangle$$

$$\frac{\partial}{\partial t}\langle\Psi^{(I)}(t)|e\rangle = -\frac{i}{\hbar}\langle\Psi(\overset{=0}{\tau})|\hat{\mu}^{(I)}(t)|e\rangle E(t)$$

Integrování Schr. r.

$$\langle e|\Psi^{(I)}(t)\rangle = \frac{i}{\hbar}\int_0^t d\tau\, \langle e|\hat{\mu}^{(I)}(\tau)|\Psi^{(I)}(\overset{=0}{\tau})\rangle E(\tau)$$

z důvodů nutnosti linearity $\Rightarrow$ omezujeme se na 1. řád teorie poruch

Počáteční podmínka

$$|\Psi(0)\rangle = |g\rangle|\Theta_g\rangle$$

stav okolí, lázně (charakterizováno teplotou T)

stav systému

Časová změna obsazení excitovaného stavu

$$\frac{\partial}{\partial t}P_e = \frac{2}{\hbar^2}|d_{eg}|^2\,\mathrm{Re}\int_0^t d\tau\, \langle\Theta_g|\tilde{U}_g^{+}(t)\,\tilde{U}_e(t)$$

$$\times\,\tilde{U}_e^{+}(\tau)\,U_g(\tau)\,|\Theta_g\rangle\, e^{-i\omega_{eg}(t-\tau)}\,E(t)E(\tau)$$

Převod do známého tvaru

$$\hat{W}_{eq} = |\Theta_g\rangle\langle\Theta_g|$$

$$\langle\Theta_g|\tilde{U}_g^{\dagger}(t)\tilde{U}_e(t)\tilde{U}_e^{\dagger}(\tau)\tilde{U}_g(\tau)|\Theta_g\rangle =$$

$$= Tr_B\left\{\tilde{U}_g^{\dagger}(t-\tau)\,\tilde{U}_e(t-\tau)\,\hat{W}_{eq}\right\} = e^{-g(t-\tau)}$$

substituce $t-\tau \to \tau$

$$\boxed{\frac{\partial}{\partial t}P_e = \frac{2}{\hbar^2}|d_{eg}|^2\, \mathrm{Re}\int_0^t d\tau\, e^{-g(\tau)-i\omega_{eg}\tau}E(t)E(t-\tau)}$$

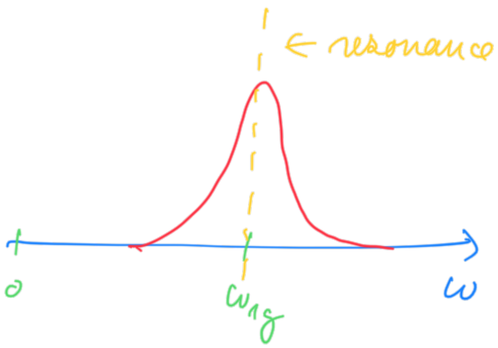
$t \to \infty$

$$E(t) = E_0\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$\int_0^t d\tau\, e^{-g(\tau)-i\omega_{eg}\tau}E(t)E(t-\tau) \approx$$

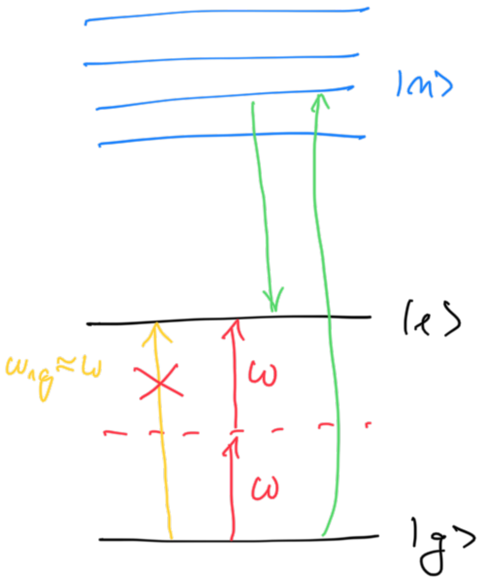
$$\approx \int_0^\infty d\tau\, e^{-g(\tau)-i(\omega_{eg}-\omega)\tau}E_0^2$$

$$\boxed{\frac{\partial}{\partial t}P_e = \frac{1}{\hbar^2}|d_{eg}|^2\, G(\omega-\omega_{eg})\,E_0^2}$$



Existuje jen jedna rezonanční podmínka ve obecném 2-hladinovém systému.

Rezonanční podmínky pro dvoufotonovou absorpci



$$\hat{H}_2 = -\hat{\mu}_2 E^2(t)$$

$$e^{i2\omega t}$$

$$\omega_{eg} \approx 2\omega$$

Dipólové momenty

$$d_{eg} = 0$$

$$d_{ng} \neq 0$$

$$d_{ne} \neq 0$$

$$\hat{H}_S = \varepsilon_g |g\rangle\langle g| + \varepsilon_e |e\rangle\langle e| + \sum_n \varepsilon_n |n\rangle\langle n|$$

$$\hat{\mu} = \sum_n \left(d_{ng} |n\rangle\langle g| + d_{gn} |g\rangle\langle n| + d_{ne} |n\rangle\langle e| + d_{en} |e\rangle\langle n| \right)$$

Efektivní pohybové rovnice pro relevantní část systému

Projekční operátor:

$$\hat{P} = |e\rangle\langle e| + |g\rangle\langle g|$$

$$\hat{1} = \underbrace{|e\rangle\langle e| + |g\rangle\langle g|}_{\hat{P}} + \underbrace{\sum_n |n\rangle\langle n|}_{\hat{Q}}$$

$$\hat{P} + \hat{Q} = \hat{1}$$

$\hat{P}\hat{H}_S\hat{P}$ ← relevantní část hamiltoniánu

$\hat{Q}\hat{H}_S\hat{Q}$ ← irelevantní — " —

Důležité vlastnosti

$$\hat{P}\hat{H}_S\hat{Q} = \hat{Q}\hat{H}_S\hat{P} = 0$$

$$\hat{P}\hat{\vec{\mu}}\hat{P} = \hat{Q}\hat{\vec{\mu}}\hat{Q} = 0$$

$$\hat{P}\hat{\vec{\mu}}\hat{Q} \neq 0$$

I.) $\frac{\partial}{\partial t}\hat{P}|\Psi^{(I)}(t)\rangle = \frac{i}{\hbar}\hat{P}\hat{\vec{\mu}}^{(I)}(\hat{P}+\hat{Q})|\Psi^{(I)}(t)\rangle E(t)$

II.) $\frac{\partial}{\partial t}\hat{Q}|\Psi^{(I)}(t)\rangle = \frac{i}{\hbar}\hat{Q}\hat{\vec{\mu}}^{(I)}(t)\hat{P}|\Psi^{(I)}(t)\rangle E(t)$

integrací II.

$$\hat{Q}|\Psi^{(I)}(t)\rangle = \frac{i}{\hbar}\int_0^t d\tau\, \hat{Q}\hat{\vec{\mu}}^{(I)}(\tau)\hat{P}|\Psi^{(I)}(\tau)\rangle E(\tau)$$

vložím do I.

$$\boxed{\frac{\partial}{\partial t}P|\Psi^{(I)}(t)\rangle = -\frac{1}{\hbar^2}\hat{P}\hat{\vec{\mu}}^{(I)}(t)\int_0^t d\tau\, \hat{Q}\hat{\vec{\mu}}^{(I)}(\tau)\hat{P}|\Psi^{(I)}(\tau)\rangle E(t)E(\tau)}$$

zajímá nás obsazení $|e\rangle$ $\rightarrow$ vynásobíme z leva $\langle e|$

$$\frac{\partial}{\partial t}\underline{\langle e|\psi^{(I)}(t)\rangle} = -\frac{1}{\hbar^2}\int_0^t d\tau \sum_n \langle e|\tilde{U}_e^+(t)\, d_{en}\, \tilde{U}_n(t)|n\rangle\langle n|$$
$$\times \tilde{U}_n^+(\tau)\, d_{ng}\, \tilde{U}_g(\tau)|g\rangle$$
$$\times \langle g|\psi^{(I)}(\tau)\rangle\, e^{i\omega_{en}t + i\omega_{ng}\tau}$$
$$\times \underline{E}(t)\underline{E}(\tau)$$

Fenomenologický popis lázně

$$\frac{\partial}{\partial t}\langle e|\psi^{(I)}(t)\rangle = -\frac{1}{\hbar^2}\sum_n d_{en}d_{ng}\, e^{+i\omega_{en}t - \gamma t} E_0^2\left(e^{i\omega t} + e^{-i\omega t}\right)$$
$$\times \underbrace{\int_0^t d\tau\left(e^{i(\omega+\omega_{ng})\tau - \gamma t} + e^{-i(\omega-\omega_{ng})\tau - \gamma t}\right)}\langle g|\psi^{(I)}(\tau)\rangle$$

$|\psi^{(I)}(\tau)\rangle$
$\approx |\psi^{(I)}(t)\rangle$

$$\boxed{\frac{\partial}{\partial t}\langle e|\psi^{(I)}(t)\rangle = -\frac{1}{\hbar^2}\sum_n d_{en}d_{ng}E_0^2 F(t)\langle g|\psi^{(I)}(t)\rangle}$$

$$F(t) = \frac{e^{i2\omega t + i\omega_{ng}t} + e^{i\omega_{ng}t} - e^{i(\omega+\omega_{en})t - \gamma t} - e^{-i(\omega-\omega_{en})t - \gamma t}}{i(\omega+\omega_{ng}) + \gamma}$$
$$+ \frac{e^{-i2\omega t + i\omega_{ng}t} + e^{i\omega_{ng}t} - e^{i(\omega+\omega_{en})t - \gamma t} - e^{-i(\omega-\omega_{en})t - \gamma t}}{i(\omega-\omega_{ng}) + \gamma}$$

$$t \to \infty \qquad e^{-\gamma t} = 0$$

$$|\omega \pm \omega_{ng}| \gg \gamma$$

$$F(t) \simeq \frac{e^{i2\omega t + i\omega_{ng} t} + e^{i\omega_{ng} t}}{i(\omega + \omega_{ng})} + \frac{e^{-i2\omega t + i\omega_{eg} t} + e^{i\omega_{ng} t}}{i(\omega - \omega_{ng})}$$

$$\boxed{2\omega = \omega_{eg}} \Rightarrow F(t) \approx \text{const.}$$

$$\frac{\partial}{\partial t} \langle e | \psi^{(I)}(t) \rangle \approx F(t) \, \langle g | \psi^{(I)}(t) \rangle$$

$$\sim c_e(t) \qquad\qquad \sim c_g(t) \approx c_g$$

$$P_e(t) = |c_e(t)|^2$$

$$\boxed{\frac{\partial}{\partial t} \langle e | \psi^{(I)}(t) \rangle = \frac{i}{\hbar} \sum_n \frac{d_{en} d_{ng}}{\hbar(\omega - \omega_{ng})} e^{i\omega_{eg} t} \langle g | \psi^{(I)}(t) \rangle E_0^2 e^{-i2\omega t}}$$

$$\hat{\mu}_{2phot} = d_{eg}^{(2)} \left(|e\rangle\langle g| + |g\rangle\langle e| \right)$$

$$\boxed{d_{eg}^{(2)} = \sum_n \frac{d_{en} d_{ng}}{\hbar(\omega - \omega_{ng})}} \xrightarrow{2\omega = \omega_{eg}} \boxed{= -\sum_n \frac{2 d_{en} d_{ng}}{\hbar(\omega_{ne} + \omega_{ng})}}$$

$$\boxed{\hat{H}_{2phot} = -\hat{\mu}_{2phot}\left(\underline{E}(t)\right)^2}$$