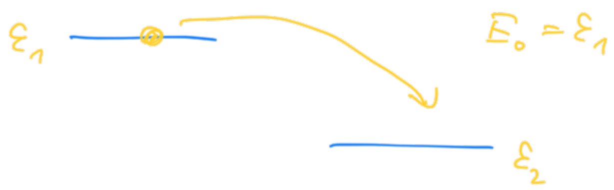
Energie a „hybnost" dvojhladinového systému

Energie při časovém vývoji

$$E(t) = \langle \psi(t)|\hat{H}|\psi(t)\rangle = \langle \psi_0|\hat{U}^\dagger(t)\,\hat{H}\,U(t)|\psi_0\rangle = \langle \psi_0|\hat{H}|\psi_0\rangle$$

$$U(t) = e^{-\frac{i}{\hbar}\hat{H}t} \qquad = E_0$$



$$H = \begin{pmatrix} \varepsilon_1 & J \\ J & \varepsilon_2 \end{pmatrix}$$

$$|\psi(t)\rangle = a(t)|0\rangle + b(t)|1\rangle$$

$$E = \left(a^*\langle 0| + b^*\langle 1|\right)\left(\varepsilon_1|0\rangle\langle 0| + \varepsilon_2|1\rangle\langle 1| + J\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)\right) \times \left(a|0\rangle + b|1\rangle\right)$$

$$= \left(a^*\varepsilon_1\langle 0| + a^* J\langle 1| + b^*\varepsilon_2\langle 1| + b^* J\langle 0|\right)\left(a|0\rangle + b|1\rangle\right)$$

$$= |a|^2\varepsilon_1 + b^* a J + a^* b J + |b|^2\varepsilon_2$$

$$E = |a|^2\varepsilon_1 + |b|^2\varepsilon_2 + J\,2\,\mathrm{Re}\,a^* b \qquad a = |a|e^{i\varphi_a} \quad b = |b|e^{i\varphi_b}$$

$$\varphi = \varphi_b - \varphi_a$$

$$E = |a|^2\varepsilon_1 + |b|^2\varepsilon_2 + 2J|a||b|\cos\varphi$$

Hybnost ve 2 hladinovém systému

$$\hat{Q} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \langle \hat{Q}\rangle = \langle \psi_0|\hat{U}^\dagger(t)\,Q\,U(t)|\psi_0\rangle$$

$$\frac{d}{dt}\langle \hat{Q}\rangle = ? \qquad \overset{???}{=} \frac{\langle \hat{p}\rangle}{m}$$

$$\frac{d}{dt}\langle \hat{Q}\rangle = \langle \psi_0 | \frac{i}{\hbar} \hat{U}^\dagger(t)\, \hat{H}\, \hat{Q}\, U(t) | \psi_0 \rangle + \langle \psi_0 | U^\dagger(t)\, Q \left(-\frac{i}{\hbar}\right) H U(t) | \psi_0 \rangle$$

$$= -\frac{i}{\hbar}\Big(\langle \psi(t) | -\hat{H}\hat{Q} + \hat{Q} H | \psi(t) \rangle\Big) = -\frac{i}{\hbar}\langle [\hat{Q}, \hat{H}] \rangle$$

$$QH = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \varepsilon_1 & J \\ J & \varepsilon_2 \end{pmatrix} = \begin{pmatrix} -\varepsilon_1 & -J \\ J & \varepsilon_2 \end{pmatrix}$$

$$HQ = \begin{pmatrix} \varepsilon_1 & J \\ J & \varepsilon_2 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\varepsilon_1 & J \\ -J & \varepsilon_2 \end{pmatrix}$$

$$QH - HQ = \begin{pmatrix} 0 & -2J \\ 2J & 0 \end{pmatrix}$$

$$\frac{d}{dt}\langle \hat{Q} \rangle = \langle \psi | \begin{pmatrix} 0 & i\frac{2J}{\hbar} \\ -i\frac{2J}{\hbar} & 0 \end{pmatrix} | \psi \rangle \qquad \boxed{\hat{p} = m\frac{2J}{\hbar}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}}$$

Meze populačních oscilací

$$|b_0|^2 = 1 \qquad P_1 = 1$$

$$\varepsilon_1 = 0\, ; \ \varepsilon_2 = \varepsilon$$

$$|b|^2 \varepsilon + 2J|a||b|\cos\varphi = \varepsilon \qquad |a| = \sqrt{1-|b|^2}$$

$$\cos\varphi = \frac{\varepsilon(1-|b|^2)}{2J|a||b|}$$

$$\cos\varphi = \frac{\varepsilon}{2J}\frac{1-|b|^2}{\sqrt{1-|b|^2}\,|b|} = \frac{\varepsilon}{2J}\sqrt{\frac{1-P_2}{P_2}}$$

Extrém $\cos\varphi = 1$

$$\frac{2J}{\varepsilon} = \sqrt{\frac{1-P_2}{P_2}} \ \Big|^2 \Rightarrow \left(\frac{2J}{\varepsilon}\right)^2 = \frac{1-P_2}{P_2}$$

$$P_2\left(\frac{2J}{\varepsilon}\right)^2 = 1 - P_2$$

$$\boxed{P_2 = \frac{1}{1+\left(\frac{2J}{\varepsilon}\right)^2}}$$