Formalismus kvantové teorie pro více systémů

$$
\begin{array}{ll}
\text{———}\ |1\rangle_1 & \text{———}\ |1\rangle_2 \\[2em]
\text{———}\ |0\rangle_1 & \text{———}\ |0\rangle_2
\end{array}
$$

$\boxed{\text{Qbit}}$

NOT

$$|1\rangle \to |0\rangle \qquad F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$|0\rangle \to |1\rangle$$

$$F^{-1} = F = F^T = F^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Hadamardovo hradlo

$$|0\rangle \to \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \to \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\langle 0| + \frac{1}{2}\left(|0\rangle - |1\rangle\right)\langle 1|$$

$$= \frac{1}{\sqrt{2}}|0\rangle\langle 0| + \frac{1}{\sqrt{2}}|1\rangle\langle 0| + \frac{1}{\sqrt{2}}|0\rangle\langle 1| - \frac{1}{\sqrt{2}}|1\rangle\langle 1|$$

$E = E_1 + E_2 \qquad |\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$

$\langle H\rangle = \langle H_1\rangle + \langle H_2\rangle \qquad H = H_1 + H_2$

$$\langle H\rangle = \langle\psi|H_1 + H_2|\psi\rangle = \left(\langle\psi_1| + \langle\psi_2|\right)\left(H_1 + H_2\right)\left(|\psi_1\rangle + |\psi_2\rangle\right)$$

$$= \underbrace{\langle\psi_1|H_1|\psi_1\rangle}_{E_1} + \underbrace{\langle\psi_2|H_2|\psi_2\rangle}_{E_2} + \overset{?}{\langle\psi_1|H_1|\psi_2\rangle}$$

$$+\langle \psi_1 | H_2 | \psi_1 \rangle$$
$$H_2$$

$$|\psi\rangle = |\psi_1\rangle |\psi_2\rangle$$

$$H = H_1 \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes H_2$$

$$\langle H \rangle = \langle \psi_1 | \langle \psi_2 | \left(H_1 \otimes \mathbb{1}_2 + \mathbb{1}_1 \otimes H_2 \right) \times |\psi_1\rangle |\psi_2\rangle$$
$$= \langle \psi_1 | H_1 | \psi_1 \rangle \langle \psi_2 | \psi_2 \rangle +$$
$$+ \langle \psi_1 | \psi_1 \rangle \langle \psi_2 | H_2 | \psi_2 \rangle$$
$$= E_1 + E_2$$

$|0\rangle_1 |0\rangle_2$
$|1\rangle_1 |0\rangle_2$
$|0\rangle_1 |1\rangle_2$
$|1\rangle_1 |1\rangle_2$

NOT na druhý Qbit

$$|0\rangle_1 |0\rangle_2 \to |0\rangle_1 |1\rangle_2$$
$$|1\rangle_1 |0\rangle_2 \to |1\rangle_1 |1\rangle_2$$
$$|0\rangle_1 |1\rangle_2 \to |0\rangle_1 |0\rangle_2$$
$$|1\rangle_1 |1\rangle_2 \to |1\rangle_1 |0\rangle_2$$

$$\begin{pmatrix}1&0\\0&1\end{pmatrix} \to \begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$\boxed{NOT_2 = \left(\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)}$$

Hadamard na 2. qbit

$$|0\rangle|0\rangle \to |0\rangle \frac{1}{\sqrt2}(|0\rangle + |1\rangle) = \frac{1}{\sqrt2}|0\rangle|0\rangle + \frac{1}{\sqrt2}|0\rangle|1\rangle$$

$$|0\rangle|1\rangle \to |0\rangle \frac{1}{\sqrt2}(|0\rangle - |1\rangle) = \frac{1}{\sqrt2}|0\rangle|0\rangle - \frac{1}{\sqrt2}|0\rangle|1\rangle$$

$$|1\rangle|0\rangle \to |1\rangle \frac{1}{\sqrt2}(|0\rangle + |1\rangle) = \frac{1}{\sqrt2}|1\rangle|0\rangle + \frac{1}{\sqrt2}|1\rangle|1\rangle$$

$$|1\rangle|1\rangle \to |1\rangle \frac{1}{\sqrt2}(|0\rangle - |1\rangle) = \text{—||—} \quad - \quad \text{—||—}$$

$$H = \frac{1}{\sqrt2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$|0\rangle|0\rangle \to |0\rangle|1\rangle$$
$$|0\rangle|1\rangle \to |0\rangle|0\rangle$$
$$|1\rangle|0\rangle \to |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \to |1\rangle|0\rangle$$

$$NOT_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Operace na dvou qbitech a měření

$$NOT = X = F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{aligned} |0\rangle &\to |1\rangle \\ |1\rangle &\to |0\rangle \end{aligned}$$

Controlled NOT = CNOT

Je-li 2. qbit $|0\rangle$ pak neprovedeme nic

—||— $|1\rangle \Rightarrow$ provedeme NOT na 1. qbitu

$$|0\rangle|0\rangle \to |0\rangle|0\rangle$$
$$|1\rangle|0\rangle \to |1\rangle|0\rangle$$
$$|0\rangle|1\rangle \to |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \to |0\rangle|1\rangle$$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Př:

$$(CNOT)(CNOT) = \left(\begin{array}{c|c} & \\ \hline & \end{array}\right)\left(\begin{array}{c|c} & \\ \hline & \end{array}\right) = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}\right) = \mathbb{1}$$

Př:

$$H_1 \otimes \mathbb{1}_2 = ?$$

Měření

$|1\rangle\langle 1|$
$|2\rangle\langle 2|$
$|3\rangle\langle 3|$

$|5\rangle\langle 5|$ $\quad \sum_n |n\rangle\langle n| = \mathbb{1}$

$|5\rangle\langle 5|\psi\rangle = a_5|5\rangle$

$|a_5|^2 = p_5$

$$|\psi\rangle = a|1\rangle + b|0\rangle$$

$$|1\rangle\langle 1| \; ; \; |2\rangle\langle 2| \quad \Rightarrow \quad \mathbb{1} = |1\rangle\langle 1| + |2\rangle\langle 2|$$

$|\psi_{nový}\rangle = |5\rangle$

$$M_5 \approx |5\rangle\langle 5|$$

$$M_5|\psi\rangle \longrightarrow \frac{1}{a_5} a_5 |5\rangle$$

Redukce, kolaps vlnové funkce.

$$|\psi\rangle \xrightarrow{\text{měření}} p_5 \; |5\rangle \qquad |\psi\rangle$$

Hrátky s qbity a kvantovým počítačem

1 qbit

$$NOT \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{aligned} |0\rangle &\to |1\rangle \\ |1\rangle &\to |0\rangle \end{aligned}$$

$$H \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \qquad \begin{aligned}|0\rangle &\to \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\\ |1\rangle &\to \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\end{aligned}$$

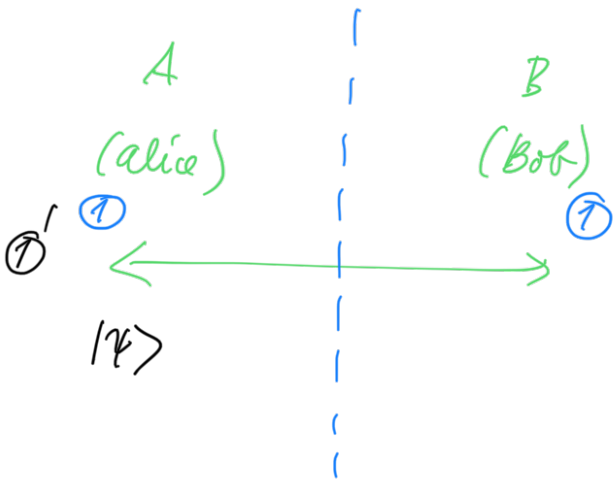
2 qbity

CNOT

$$|\psi(t)\rangle \neq |\psi_1\rangle|\psi_2\rangle \qquad \frac{|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2}{\sqrt{2}}$$

Entanglement ← quantum entanglement
kvantové provázání

$$|\beta\rangle = \frac{|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2}{\sqrt{2}}$$



Teleportace kvantového stavu

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\phi\rangle = |\psi\rangle|\beta\rangle = \left(\alpha|0\rangle + \beta|1\rangle\right)\left(\frac{|0\rangle|0\rangle + |1\rangle|1\rangle}{\sqrt{2}}\right)$$

1) alice provede $CNOT_1$:

ve videu chybí závorka

$$|\phi\rangle_1 = \frac{1}{\sqrt{2}}\alpha|0\rangle\left(|0\rangle|0\rangle+|1\rangle|1\rangle\right) + \beta|1\rangle\left(|1\rangle|0\rangle+|0\rangle|1\rangle\right)$$

2) alice H_1:

$$|\phi\rangle_2 = \frac{1}{\sqrt{2}}\left(\alpha\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\left(|0\rangle|0\rangle+|1\rangle|1\rangle\right) + \beta\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)\times\left(|1\rangle|0\rangle+|0\rangle|1\rangle\right)\right)$$

$$= \frac{1}{2}\Big(|0\rangle|0\rangle\left(\alpha|0\rangle+\beta|1\rangle\right) + |0\rangle|1\rangle\left(\alpha|1\rangle+\beta|0\rangle\right) + |1\rangle|0\rangle\left(\alpha|0\rangle-\beta|1\rangle\right) + |1\rangle|1\rangle\left(\alpha|1\rangle-\beta|0\rangle\right)\Big)$$

3) alice měření

alice změří

$$|\phi\rangle_2 \rightarrow |0\rangle|0\rangle\left(\alpha|0\rangle+\beta|1\rangle\right) = |\psi\rangle$$

$$|0\rangle|1\rangle\underbrace{\left(\alpha|1\rangle+\beta|0\rangle\right)}_{|\varphi\rangle} \Rightarrow |\psi\rangle = NOT\,|\varphi\rangle$$

Teleportace kvantového stavu

Zákaz klonování kvantového stavu

A

B

1'

1

2

$|\psi\rangle_{1'}$

CNOT

H

klasická informace

$|\psi_2\rangle$

$$|\phi\rangle = \frac{1}{2}\Big(|0\rangle|0\rangle\big(\alpha|0\rangle + \beta|1\rangle\big) + |1\rangle|0\rangle\big(\alpha|0\rangle - \beta|1\rangle\big)$$
$$+ |0\rangle|1\rangle\big(\alpha|1\rangle + \beta|0\rangle\big) + |1\rangle|1\rangle\big(\alpha|1\rangle - \beta|0\rangle\big)\Big)$$

měření

$$|\phi\rangle \xrightarrow{\text{redukuje}} \underline{|0\rangle|0\rangle}\big(\alpha|0\rangle + \beta|1\rangle\big)$$

Je možné klonovat?

$$|0\rangle_1|\psi\rangle_2 \xrightarrow{\hat{C}} |\psi\rangle_1|\psi\rangle_2$$

není možné klonovat

Důkaz:

$$\hat{C}\,|0\rangle_1|\psi_1\rangle_2 = |\psi_1\rangle_1|\psi_1\rangle_2$$

$$\hat{C}\,|0\rangle_1|\psi_2\rangle_2 = |\psi_2\rangle_1|\psi_2\rangle_2$$

Skalární součin

$${}_2\langle\psi_1|\,{}_1\langle 0|\,\hat{C}^{+}\hat{C}\,|0\rangle_1|\psi_2\rangle_2$$

$$= \langle\psi_1|\langle\psi_1|\psi_2\rangle|\psi_2\rangle = |\langle\psi_1|\psi_2\rangle|^2$$

$${}_2\langle\psi_1|\,{}_1\langle 0|0\rangle_1|\psi_2\rangle_2 = \langle\psi_1|\psi_2\rangle$$

ve videu je chybně index

$$|\langle\psi_1|\psi_2\rangle|^2 = \langle\psi_1|\psi_2\rangle$$

$$x^2 = x \quad\Rightarrow\quad x = 0 \qquad |\psi_1\rangle \perp |\psi_2\rangle$$
$$\qquad\qquad\quad\; x = 1 \qquad |\psi_1\rangle = |\psi_2\rangle$$